The Edmonton Junior High Team Math Contest

Group member #1 first name, surname, school:

____

Group member #2 first name, surname, school:

____

Group member #3 first name, surname, school:

____

Group member #4 first name, surname, school:

____

Rules:

1) Teams up to 4 people.
2) You can use a protractor, compass, ruler, as with any writing utensils (except red pen).
3) No calculators allowed.
4) You will be given 1.5 hours to complete the contest.
5) Only legible writing will be marked.
6) Outline
    a) Part A is comprised of 10 questions, with each question worth 4 points. Only the answer is required, however partial marks may be given for work shown.
    b) Part B is comprised of 6 questions, with each question worth 10 points. YOU MUST SHOW ALL YOUR WORK FOR FULL MARKS.

PLEASE PUT THIS SHEET INSIDE THE BOOKLET WHEN YOU ARE HANDING IN.

PART A

Problem 1 Everything in the world is at least two of

1. i. A dolphin
2. ii. A mathematician
3. iii. A tiger

A dolphin can not also be a tiger. Each group is very direct, it is known that either their entire group hates pianos, or their entire group likes pianos. If all dolphins hate pianos, is it necessarily true that all tigers also hate pianos?

Problem 2 Evaluate

$$(-100^{-100})(-99^{-99})...(99^{99})(100^{100}) \tag{1}$$

Problem 3 In a game Ralph's last game of ballbasket, he scored a total of 30 points. His score was a combination of shots that were worth 2 point and 3 points. If Ralph did **not** score more 3 pointers than 2 pointers, how many two pointers and 3 pointers did he score?

Problem 4 There are 10 cards in the deck, 8 red and 2 blue. Ralph and Oscar are dealt 5 cards at random. They each pick a random three cards from the deck and place it onto the board. What is the probability that the 6 cards on the board are all red?

Problem 5 A beam of light is shot at a 45 degree angle from a flat surface. Two mirrors are set up parallel to each other and perpendicular to the surface such that the beam of light - in order - passes under the first mirror, hits the second and then goes above the first mirror again. If the mirrors are 3 metres apart, what is the maximum length of the first mirror?

Problem 6 Joseph and Ron run on a circular track of length 5 km. It is known Joseph and Ron run clockwise at a constant speed of 6 km/h and 9 km/h respectively and start at the same point. Let the next point Joseph and Ron meet be n km clockwise from their starting point. Find n.

Problem 7 A pandigital number is a positive integer such that all digits are included (not necessarily only once). For example, 442018935867 is a pandigital number in base-10, as it uses all digits from 0-9. Find the smallest integer such that it is pandigital when expressed in base-3 and in base-4. Write your answer in base-10

Addendum: A base-b number $\overline{a_1 a_2 ... a_n}$ can be expressed as $a_1 b^{n-1} + a_2 b^{n-2} + ... + a_n b^0$

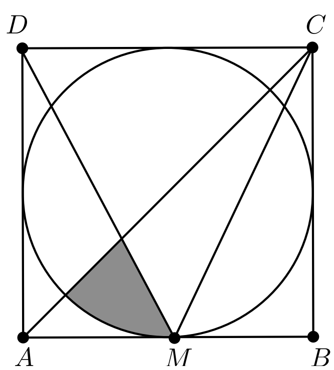
Problem 8 We define the new function Ξ, such that

$$x \,\Xi\, y = \begin{cases} x \text{ if } x \text{ and } y \text{ are coprime} \\ x + y \text{ if } x \text{ and } y \text{ are not coprime.} \end{cases}$$

Two number are coprime if they have no common factors other than 1.

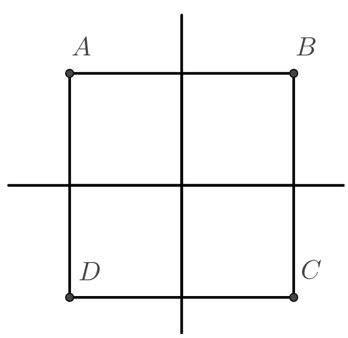
Evaluate 2 Ξ (3 Ξ (...(99 Ξ 100))...)

Problem 9 Let ABCD be a square with point M be the midpoint of side AB. Inscribe a circle of center O in the square. Let N be the intersection of lines AC and MD. Find the ratio of the area of the shaded region to the area of the square.



Problem 10 Cut a square with side length 4 into four distinct non-overlapping parts with two straight lines such that the area of the four parts make the ratio 1:1:2:4. Label the length of all new sides of the square.

Example of using two lines to cut the square into a 1:1:1:1 ratio



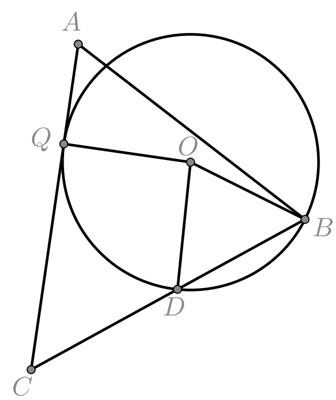
PART B

Problem 1 Given that x, y are positive integers and $x \neq -y, -2y, -3y, -4y$. Find which expression always outputs a larger value

$$\frac{1}{\frac{4}{x+3y} - \frac{4}{x+4y}} - \frac{1}{\frac{4}{x+y} - \frac{4}{x+2y}} \qquad or \qquad x+2y$$

Problem 2 Jack wants to line up 7 books on a bookshelf cover to cover The width of the books are 1 unit wide, if the length of the books are $2, 4, 8, ..., 2^7$. Perimeter made up of the spines of the books. Out of all possible arrangements what is the largest perimeter Jack can create?

Problem 3 Given ΔABC with D on BC. Let the circle with centre O be tangent to AC at Q and pass through both B and D. Prove that $\angle ACB = \angle QDO$ if and only if O is on line BQ.



Problem 4 A 4x4 board can be filled with M, A, T, and H. Chuck the Chomper enjoys chomping on M, A, T, and H's. However, Chomp will only binge chomp, so he will only chomp on M, A, T, and H's when there is both an entire row and column of M, A, T, and H's. If a square has already been chomped on, Chuck won't acknowledge its existence. If the board is initially filled out as done in the diagram below, how many different ways can you finish tiling the 4×4 board such that all M, A, T, and H's will be chomped on by the end?

H			
T			
A			
M	A	T	H

Problem 5 A unidigital number is one that is constructed out of only one digit. Find the number of 2's in a unidigital number made exclusively out of 2's and is divisible by 2018. (Note: There are several solutions, but only one final answer) (Source: Canada)

Problem 6 Ralph only reads prime numbered pages. Oscar only reads composite numbered pages. Starting from 1, what is the maximum number of pages in the book such that the number of pages Oscar reads is exactly double the number of pages Ralph reads?