2023 Team Math Attack Contest

Team solutions

December 9, 2023

Answers

1. 17
2. 20
3. 2/3
4. 19
5. 120
6. 3/32
7. 1
8. $\pi/4$
9. 64.5
10. 10
11. 6/11
12. 5
13. 7
14. 20
15. 46
16. 120/3
17. 180
18. 1/4
19. 2
20. 172

Solutions

1. **Answer:** $\boxed{17}$
   $2 \times 2 + 0 \times 0 + 2 \times 2 + 3 \times 3 = 4 + 0 + 4 + 9 = 17$

2. **Answer:** $\boxed{20}$
   If Anna sells 7 cookies for 5 dollars, then if she sells $28 = 7 \times 4$ cookies she will obtain $5 \times 4 = 20$ dollars.

3. **Answer:** $\boxed{2/3}$
   Calculating the perimeters of the squares we have 4, 8, 12, 16, 20, 24 respectively. There are 4 of these perimeters that are greater than 10. Thus the probability is $4/6 = 2/3$.

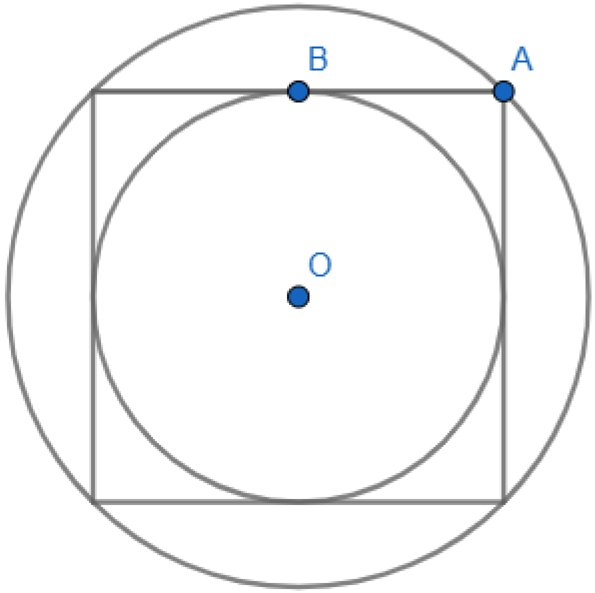
4. **Answer:** $\boxed{19}$
   The only possibility for the three different integers are 1, 7, 11. $1 + 7 + 11 = 19$.

5. **Answer:** $\boxed{120}$
   Let the number of votes be x. Donald won $0.6x$ votes, while Barry won $0.4x$ votes. The difference in their votes is $0.6x - 0.4x = 0.2x = 24$. Therefore, $x = 24/0.2 = 120$.

6. **Answer:** $\boxed{3/32}$
   The only arrangements that satisfy the conditions are HHHTT, THHHT, and TTHHH. There are a total of $2^5 = 32$ possible arrangements. Thus, the probability is $3/32$.

7. **Answer:** $\boxed{1}$
   The pattern above has a period of 6. 2023 has a remainder of 1 when divided by 6. Thus, by simply observing the pattern, we see that every digit that has a remainder of 1 when divided by 6 has a value of 1. Therefore, the 2023rd digit is 1.

8. **Answer:** $\boxed{\pi/4}$
   Drawing a diagram,



Since the side length of the square is 1, we can find out the radius of each of the circles. The radius of circle B, OB, is simply half the side length of the square which is 1/2. The radius of circle A, OA, can be found using Pythagorean theorem since $OB \perp AB$. $OA = \sqrt{(1/2)^2 + (1/2)^2} = 1/\sqrt{2}$. Then, we can find the areas of each of the circles using the $A = \pi r^2$ formula. The area of circle A is $\pi/2$ while the area of circle B is $\pi/4$, thus $a - b = \pi/2 - \pi/4 = \pi/4$.

9. **Answer:** $\boxed{64.5}$

   The arithmetic sequence can either be 20, 23, x ; 20, x, 23 ; or x, 20, 23. In the first case, the common difference is $23 - 20 = 3$, thus $x = 23 + 3 = 26$. In the second case, the common difference is $(23 - 20)/2 = 1.5$, thus $x = 20 + 1.5 + 21.5$. In the third case, the common difference is also 3, thus $x = 20 - 3 = 17$. $26 + 21.5 + 17 = 64.5$.

10. **Answer:** $\boxed{10}$

    Erik and his 4 besties can finish $300/20 = 15$ problems per day. Thus, in 15 days, they will finish $15 \cdot 15 = 225$ problems. That means the other members of MAS finished $300 - 225 = 75$ problems in 15 days, or $75/15 = 5$ problems per day. Thus, there are $5 + 5 = 10$ people at MAS.

11. **Answer:** $\boxed{6/11}$

    This is a standard problem of conditional probability. Since it asks for the probability that it rained with the condition that Jack went to the park, first the probability that Jack goes to the park must be determined. This is equal to $60\% \cdot 40\% + 40\% \cdot 50\% = 44\%$ (probability that it rains and Jack goes to the park added with the probability that it doesn't rain and Jack goes to the park). Then we determine the probability that it rains and Jack goes to the park, which is $60\% \cdot 40\% = 24\%$. Thus, the probability of it raining on a day that Jack goes to the park is $24\%/44\% = 6/11$

12. **Answer:** $\boxed{5}$

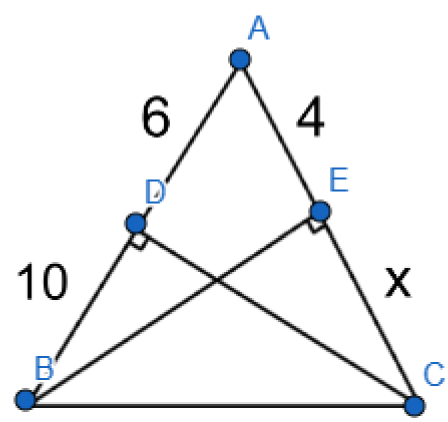
    We can first utilize the fact that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$. Thus, $x = 2^{2024} - 1$. Then, we can find a pattern in the units digit of the powers of 2. Note that $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$, $2^8 = 256$, $2^9 = 512$. It is quite clear that the units digit of the powers of 2 has a pattern with period 4, of 2,4,6,8. Thus, we can determine that the units digit of 2^2024 is 6, thus the units digit of x is $6 - 1 = 5$.

13. **Answer:** $\boxed{7}$

A four digit palindrome will always be in the form of $abba$. For it to be divisible by 12, the number ba must be divisible by 4, and $2(a + b)$ must be divisible by 3, which can be simplified to just $a + b$ being divisible by 3. Thus, listing out all combinations of ba, we have 12,24,36,48,60,72,84, and 96. However, 60 must be excluded since it would result in the first digit being 0. Thus, in total there are 7 four digit palindromes that are divisible by 12.

14. **Answer:** $\boxed{20}$

Let the points be labeled as shown.



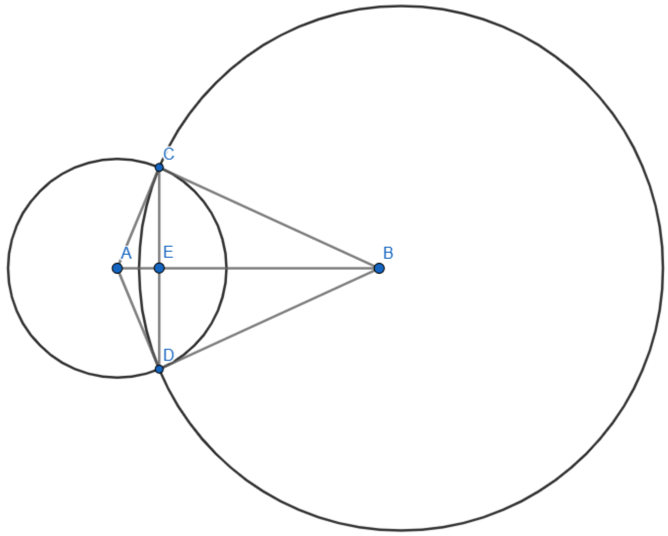
Since both triangle $\triangle ADC$ and $\triangle AEB$ are right triangles and also share $\angle A$, they are similar. Thus, we have $AC/AB = AD/AE$, and $(x + 4)/16 = 6/4$. Solving for x gives $x = 20$.

15. **Answer:** $\boxed{46}$

First, we can find the total number of possibilities without any restrictions, which is $\binom{8}{3} = 56$. Now, we should subtract the number of undesirable cases, where Jesse chooses the three sciences. In that case, there are only 2 other courses that he may choose from the remaining 5 options, i.e. $\binom{5}{2} = 10$. 56-10 $= 46$.

16. **Answer:** $\boxed{120/3}$

Drawing out a diagram, we have

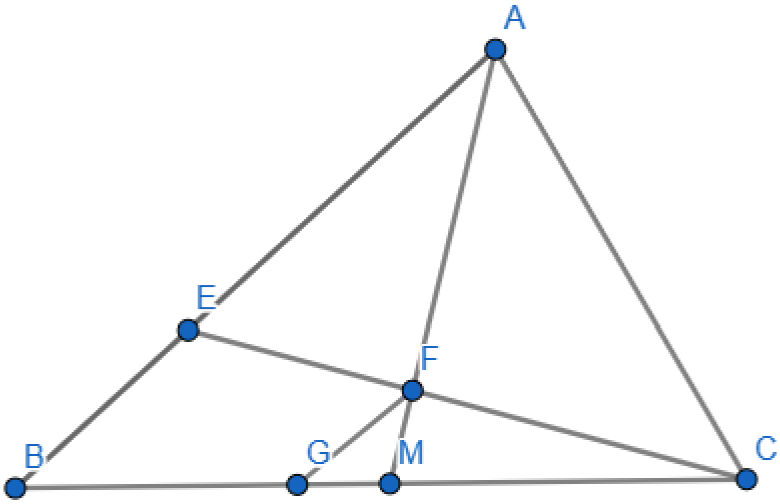


Note that since $AC = AD = 5$, $BC = BD = 12$, and $AB = 13$, triangles $\triangle ACB$ and $\triangle ADB$ are congruent right triangles, and $CD \perp AB$. Focusing on $\triangle ACB$, we want to calculate CE. Using the area of a triangle, $A = AC \cdot BC/2 = AB \cdot CE/2$, thus $AC \cdot BC = AB \cdot CE$. Solving for CE, $CE = AC \cdot BC/AB = 5 \cdot 12/13 = 60/13$. Thus, $CD = 2CE = 120/13$.

17. **Answer:** $\boxed{180}$
Let x be the number of rows and y be the number of columns. Thus, we have $(x-1)y = 168$ and $x(y-1) = 165$. Expanding the equations and subtracting the second equation from the first equation, we have $x - y = 3$, or $x = y + 3$. Substituting x into the second equation, we have $(y+3)(y-1) = 165$. Solving this quadratic will yield $y = 12$ (since we are only interested in positive solutions), and thus $x = 15$. Since the question asks for the amount of cookies in the beginning, the answer is simply $x \cdot y = 15 \cdot 12 = 180$.

18. **Answer:** $\boxed{1/4}$
Note that since each square in the coordinate plane will be the same, we can actually only focus on one square. Thus, we can determine the probability that the point is within $1/2\sqrt{\pi}$ units of distance from a lattice point for a single square, which will be equal to the probability of the entire rectangle. The area that fulfills the conditions are 4 quarter circles around each of the four vertexes, with a radius of $1/2\sqrt{\pi}$. Thus, we only need to find in total the area of 1 circle with a radius of $1/2\sqrt{\pi}$. $A = \pi r^2 = \pi(1/2\sqrt{\pi})^2 = 1/4$. Thus, the probability will simply be $(1/4)/1 = 1/4$.

19. **Answer:** $\boxed{2}$
Draw $FG \parallel AB$, where G lies on BC. Then we have the diagram shown below.



Since $CF/EF = 3/2$, then $CF/CE = 3/5$. By similar triangles $\triangle CFG$ and $\triangle CEB$, we have $CG/CB = FG/EB = CF/CE = 3/5$. Since M is the midpoint of BC, we can find $MG = CG - CM = 3BC/5 - BC/2 = BC/10$. Thus $MG/MB = (BC/10)/(BC/2) = 1/5$. Then, using similar triangles $\triangle MFG$ and $\triangle MAB$, we have $FG/AB = MG/MB = 1/5$. Thus, we can find $(FG/EB)(AB/FG) = AB/EB = 3$. Finally, we can easily find $AE/BE = (AB - BE)/EB = 3 - 1 = 2$.

20. **Answer:** $\boxed{172}$

Tackling this directly with casework will be extremely tedious. Instead, we will use complementary counting. Our approach will be to first find the number of factors strictly below 10^{10}, and then subtract the number of factors below or equal to 10^4.

Step 1: Finding the number of factors strictly below 10^{10}:

Exactly 220 of the factors are below 10^{10} because of factor pairs. To see why, first notice that 10^{10} is the square root of 10^{20}. Since all factor pairs are composed of one number that is smaller than the 10^{10} and one number that is bigger than 10^{10}, there are precisely $\frac{441-1}{2} = 220$ factors under 10^{10}.

Step 2: Finding the number of factors below or equal to 10^4:

We can split the factors of 10^{20} that are below or equal to 10^4 into two groups: 1) Those that are also factors of 10^4, and 2) those that are not.

Because all the factors of 10^4 are below or equal to 10^4, the number of elements in the first group is simply the number of factors of 10^4, which evaluates to $(4+1) \times (4+1) = 25$ by the number of factors formula.

For the second group, we will have to resort to casework. We are searching for the number of factors 10^{20} that are below or equal to 10^4, that are not factors of 10^4. Each factor of 10^{20} is equal to some power of 2 multiplied by some power of 5, so we can do casework upon the power of 5.

If the power of 5 is 0, the power of 2 can be $5, 6, 7, \cdots 11, 12, 13 = 9$ values

If the power of 5 is 1, the power of 2 can be $5, 6, 7, 8, 9, 10 = 6$ values.

If the power of 5 is 2, the power of 2 can be $5, 6, 7, 8 = 4$ values.

If the power of 5 is 3, the power of 2 can be $5, 6 = 2$ values.

If the power of 5 is 4, there is no valid power of 2.

If the power of 5 is 5, the power of 2 can be $0, 1 = 2$ values.

If the power of 5 is larger than 6, there is no valid power of 2.

Adding together the number of valid factors of 10^{20} in both groups gives: $25 + 9 + 6 + 4 + 2 + 2 = 48$ total valid factors.

<u>Step 3: Applying complementary counting</u>:
Our final answer will be the difference of the values found in Step 1 and Step 2, or $220 - 48 = 172$.