2023 Team Math Attack Contest

Relay solutions

December 9, 2023

Answers

1. 6
2. 120
3. 376
4. 1
5. 12/13
6. 2760
7. 32
8. 66
9. 13/132
10. 37
11. 420/37
12. 184
13. 1
14. 32
15. 7

Solutions

1. Answer: $\boxed{6}$
   We first solve for x in $x + 3y = 10$ in terms of y: $x = 10 - 3y$.
   We plug that into $2^x = 4^y$, where we get $2^{10-3y} = 4^y$. 4^y can be expressed as 2^{2y}. We take log of both sides to get $10 - 3y = 2y$.
   $y = 2$ and $x = 4$. Thus, $P = 6$

2. Answer: $\boxed{120}$

   There are 6! ways to arrange 6 different people in 6 different spots. However, because we are considering rotations, each arrangement can be rotated 6 times. Thus we must divide the total by 6. $\frac{6!}{6} = 120$

3. Answer: $\boxed{376}$

   $120 = 2^3 \times 5 \times 3$. Therefore, the number of factors is $(3+1)(1+1)(1+1) = 16$, by the formula for sum of factors. The sum of the factors is $(2^3 + 2^2 + 2^1 + 2^0)(5^1 + 5^0)(3^1 + 3^0) = (15)(6)(4) = 360$. The sum of these two numbers is $16 + 360 = 376$. Alternatively, you can list the factors of 120, and use that list to find the sum of factors and number of factors.

4. Answer: $\boxed{1}$

   We make the equation $\frac{16+x}{29+x} = 0.9$ If we solve this equation we get $x = 101$, thus the remainder is 1.

5. Answer: $\boxed{\frac{12}{13}}$

   Aiden's speed is $\frac{1}{12}$. Edward's speed is 1. Problems needed divided by speed of workers equals time T. We need 1 problem, so: $\frac{1}{\frac{1}{12}+1} = \frac{12}{13}$. The answer is $T = \frac{12}{13}$ hours.

6. Answer: $\boxed{2760}$

   Since the water drains out at a rate of $\frac{12}{13}$ litres per second for 26 minutes, $\frac{12}{13}(26 \cdot 60) = 1440$ litres are drained in the first 26 minutes. Then, since there is now water added at a rate of 1/2 litres per second, the new rate at which the water is drained is $\frac{12}{13} - \frac{1}{2} = \frac{11}{26}$ litres per second. Then the water drains for 52 more minutes, thus $\frac{11}{26}(52 \cdot 60) = 1320$ litres is drained. $1440 + 1320 = 2760$ litres.

7. Answer: $\boxed{32}$

   We can work through this problem in reverse. If he has 1 cat at the end, they had 6 cats before selling 5 cats, and if 6 is $1 - \frac{1}{4} = \frac{3}{4}$ of their cats before those ones ran away, they must have had 8 cats beforehand. Continuing this process, we see that we get $1 \longrightarrow 6 \longrightarrow 8 \longrightarrow 10 \longrightarrow 15 \longrightarrow 16 \longrightarrow \boxed{32}$

8. Answer: $\boxed{66}$

   We find the number of 3 digit numbers that meet the requirements, categorizing by the first digit.
   For numbers with first digit 1, there are 4 solutions: $187, 178, 196, 169$
   For numbers with first digit 2, there are 5 solutions: $277, 268, 286, 295, 259$
   We do this for every first digit from 1 to 9. The total is: $4 + 5 + 6 + 7 + 8 + 9 + 10 + 9 + 8 = 66$

9. Answer: $\boxed{\frac{13}{132}}$

   We want the product to be divisible by 3 and 11. We can look at each multiple of 11. For 11, there are 11 numbers between 1 and 33 that are divisible by 3, for 22, there are also 11 numbers divisible by 3, and for 33, any number paired with it will be divisible by 33, so there are 32 other numbers that work. However, there are repeated cases for (11, 33) and (22, 33), so we have to subtract 2 from our total. $11 + 11 + 32 - 2 = 52$. There are $(33 \times 32)/2 = 528$ ways of choosing 2 distinct numbers. $\frac{52}{528} = \boxed{\frac{13}{132}}$.

10. Answer: $\boxed{37}$

    We try every perfect square less than 100, since the problem specifies the answer must be 2 digit. Starting from the largest perfect square less than 100, which is 81.
    $81 + 1 = 82$, which is not prime

$64 + 1 = 65$, which is not prime

$49 + 1 = 50$, which is not prime

$36 + 1 = 37$, which is prime

Since we started from the largest squares and went down, we do not need to check the rest of the perfect squares, such as $25, 16$, etc. Thus 37 is the answer.

11. Answer: $\boxed{\frac{420}{37}}$

    By Pythagorean's theorem, we can calculate that $x = \sqrt{12^2 + 35^2} = 37$. Now we can express the area of the triangle in two different ways, $A = 12 \cdot 35/2 = 37 \cdot h/2$, where h is the height of the triangle of the side length of x. Thus, solving for h we have $h = 35 \cdot 12/37 = 420/37$.

12. Answer: $\boxed{184}$

    We first find p. From the problem, we know $p = m - 5n$. m is 420. n is 37. Thus $p = 420 - 185 = 235$. Next, we factorize p into $5 \cdot 47$. The integers that fulfill the problem's requirement must therefore not be a multiple of 5 or 47, and less than 235. From 1 to 235, there are 234 possible integers. Of those, 46 integers are multiples of 5, and 4 integers are multiples of 47. We know the answer should be $234 - 46 - 4 = 184$

13. Answer: $\boxed{1}$

    From the question, we know $x \equiv 2 \mod 7$. Thus, $4x \equiv (4 \cdot 2) \mod 7 \equiv 8 \mod 7 \equiv 1 \mod 7$. Therefore the answer is 1.

    Alternatively we can assign x as any value that has a remainder of 2 when divided by 7, for example 2. $4 \cdot 2$=8, and obviously 8 has a remainder of 1 when divided by 7.

14. Answer: $\boxed{32}$

    We see that this is an isosceles trapezoid. Knowing this, we can draw this with 7 as the base and draw two altitudes from the top 2 points to the side with length 7. Calculating the length of each partition of the base, we see they are 3, 1, and 3, from left to right. Using the Pythagorean theorem, we can determine the height, which is $\sqrt{5^2 - 3^2} = \sqrt{16} = 4$. From this, we can use the Pythagorean theorem to determine that the diagonals are both $\sqrt{4^2 + 4^2} = \sqrt{32}$ and that their product is $\boxed{32}$.

15. Answer: $\boxed{7}$

    By vieta's formula, the three roots must multiply to $z + 8 = 40$. Possible distinct integer roots that multiply to an absolute value of 40 are (1,2,20), (1,4,10), (1,5,8). (2,4,5). From the three roots, either all 3 are negative or 2 are positive and 1 negative. Meanwhile, A is simply the negative of the sum of the roots. Thus we need to find all possible values for the sum of the roots. For (1,2,20) summing up all cases of either 3 negative roots or 1 negative and 2 positive roots gives -23+21+19-17=0. For (1,4,10), summing all cases again gives -15+13+7-5=0. For (1,5,8), the sum of the cases is also -14+12+4-2=0. But for (2,4,5), the possibilities for the sum of roots are -11,7,3, and 1. However we already included the value of 7 in (1,4,10), thus we need to exclude it here, giving a sum of -11+3+1=-7. Thus the total sum of all the possibilities for the sum of the distinct roots is -7, which implies that the sum of all possible values for A is equal to 7.