1 Round 1

Jeremy has to head to school each day. His school is 2100m from his house. The first day, he goes by bike, at 20km/h, the next day by bus at 30km/h, and the third day by car at 60km/h.

1.1 Problem 1

How long does it take for Jeremy to get to school when he is biking? Express this answer in minutes with decimals.

ANSWER: $\boxed{6.3}$

$\frac{2.1km}{20}$km/h$=\frac{2.1}{20}$h$=\frac{6.3}{60}$ h $= 6.3$ min

1.2 Problem 2

How long does it take for Jeremy to get to school when he is bussing? Express this answer in minutes with decimals.

ANSWER: $\boxed{4.2}$

$\frac{2.1km}{30km/h} = \frac{2.1}{30}h = \frac{4.2}{60}$ h $= 4.2$ min

1.3 Problem 3

What is his average speed throughout all 3 commutes in km/h? Express this answer in km/h.

ANSWER: $\boxed{30}$

You may have tried directly averaging the speeds, but this is wrong. You have to calculate the total time taken for each transportation and use that to divide the total distance travelled. Therefore, the total time taken is equal to $\frac{2.1}{20} + \frac{2.1}{30} + \frac{2.1}{60} = \frac{12.6}{60}$ hours to go 6.3 km. This means the overall speed is $\frac{6.3}{\frac{12.6}{60}} = 30$km/h.

2 Round 2

2.1 Problem 1

Today is Thursday, September 8, 2022. What day of the week will it be 100 days from today? If it is a Monday, use $n = 1$ for the next problem. If it is a Tuesday, use $n = 2 \ldots$ If it is a Saturday, use $n = 6$. If it is a Sunday, use $n = 7$.

ANSWER: $\boxed{6}$

Through modular arithmetic, we see that 100 days is the same as 2 days and an integer number of weeks. 2 days after Thursday is Saturday.

2.2 Problem 2

A magic candle is n centimeters long, where n is the answer to the previous problem. For every cm of the candle that burns, an additional 2 cm is added to the candle. In addition, for every cm of the candle that burns, the next cm of the candle takes one minute longer to burn. Assuming that the first cm of the candle takes 1 minute to burn, in how many minutes will the candle be double its original length? Let this number be m, which we will use in the next problem

ANSWER: $\boxed{21}$

We see that for every centimeter burned, it gains a centimeter overall. Thus we just need to burn 6 centimeters. Each time we burn a centimeter it takes one minute longer to burn a centimeter and thus we need to burn the candle for $1 + 2 + 3 + 4 + 5 + 6 = 21$ minutes.

2.3 Problem 3

What is the total number of distinct rectangles with integer side lengths that have an area of m, where m is the answer to the previous problem?

ANSWER: $\boxed{2}$

There are only 2 distinct factorizations of 21 (1×21 and 3×7), so there are only 2 such rectangles.

3 Round 3

Aphelios has 5 different weapons: rifle, scythe, cannon, flamethrower, and chakram. He goes through one full cycle when he has used all his weapons once. After a cycle is completed, Aphelios returns to the first weapon used and continues to use his weapons in the same order.

Note: The order is circular, and rotations are considered the same. For example, rifle, scythe, cannon, flamethrower, and chakram are the same as scythe, cannon, flamethrower, chakram, and rifle.

3.1 Problem 1

If he uses one weapon at a time, how many weapon orders are possible?

ANSWER: $\boxed{24}$

5!/5=24

3.2 Problem 2

If he wants to use his scythe and chakram consecutively, how many weapon orders are possible?

ANSWER: $\boxed{12}$

Set scythe and chakram together as one unique entity. 4!/4=6 and then multiply by 2 because either scythe first or chakram first to get 12.

3.3 Problem 3

What is the probability that cannon and chakram are not used consecutively?

ANSWER: $\boxed{\frac{1}{2}}$

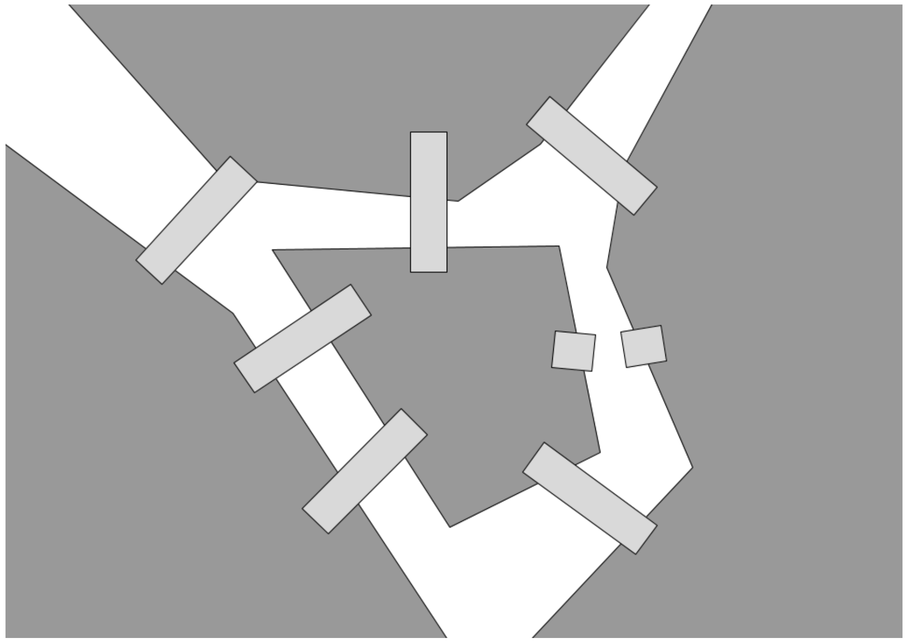
Find combinations of cannon and chakram not consecutive by subtracting combinations where they are consecutive from total combinations. 24-12=12. P=12/24=$\frac{1}{2}$.

4 Round 4

4.1 Problem 1

People have been disappointed in the infrastructure of Gammasberg for a long time. Ashachu is attempting to beat the gym challenge, which is to go across all of the bridges once. He is finding difficulty with this, so Ashachu has tactically removed a bridge, as shown below. How many ways are there for Ashachu to cross on all of the remaining bridges exactly once now if he starts from the top island?

Note: Ashachu may not swim or remove more bridges. He can move around on individual landmasses as much as he can. The landmasses are not connected outside the image.



ANSWER: $\boxed{16}$

Through careful casework, you can work out that there are 4 paths going through the left path, 6 going through the center, and 6 going throught the right path. 4 + 6 + 6 = 16

4.2 Problem 2

Let m be the answer to the previous problem. In the magical world of Mathamon, Pi and Ashachu are in a battle against Mathikyu with 100 health. Each time Ashachu attacks, Ashachu will reduce Mathakyu's health by m. After every 5th time Ashachu attacks, Mathakyu's current health will increase by exactly 50%. How many attacks does it take for Ashachu to defeat Mathakyu?

ANSWER: $\boxed{7}$

The first 5 hits will take Mathakyu down $5 \cdot 16 = 80$ health, leaving $100 - 80 = 20$ health. Mathakyu heals up to $1.5 \cdot 20 = 30$ health. Two more hits take Mathakyu down to 14 and below 0, taking a total of $5 + 2 = 7$ attacks for Ashachu to defeat Mathakyu.

4.3 Problem 3

Let n be the answer to the previous problem. Ashachu is putting his badges in holders after winning the Pokemon league. He can put his badges in holders of 6 or 11. If he puts them in holders of 6, he will have 5 badges left over, and if he puts them in holders of 11, he has n badges left over. What is the smallest number of badges he can have?

ANSWER: $\boxed{29}$

We can simply use Chinese remainder theorem or brute force to solve this. $n \equiv 5 \pmod 6$ and $n \equiv 7 \pmod{11}$ gives us a minimum number of 29 badges.

5 Round 5

Note: we noticed a small error in our contest a bit before it began, so you may have seen variations in how question 1 was changed in your city.

5.1 Problem 1

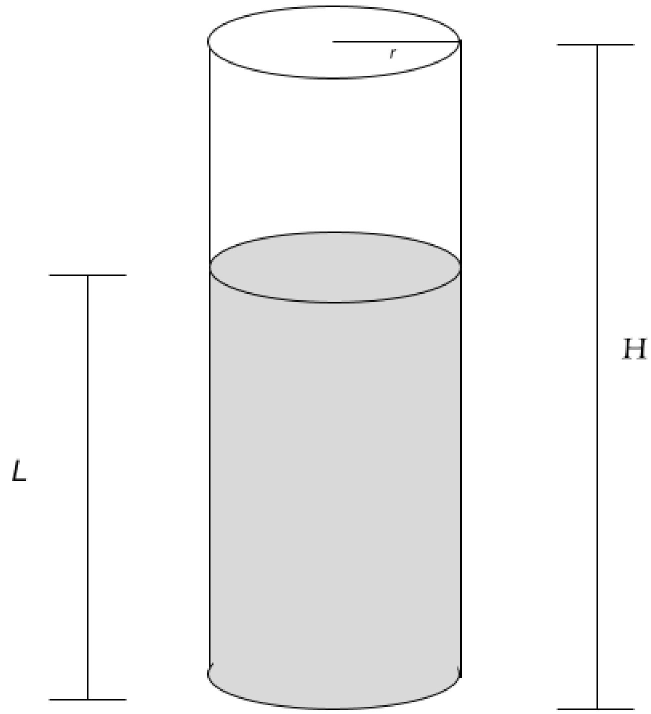
A clock strikes at regular intervals throughout the day, starting at exactly 12 o'clock. Between 1 and 2 o'clock, it strikes 6 times. What is the greatest number of times it can strike in the three hours from 1 o'clock to 4 o'clock? Let this number be V.

ANSWER: $\boxed{19}$

You would find that if the intervals were around 9.5 minutes, they would still ring 6 times between 1 and 2 and ring 19 times in total between 1 and 4.

5.2 Problem 2

Ryan pours an unknown amount of a mystery liquid into an upright cylindrical bottle. Assume that the liquid's volume is $V\pi$ cubic units, where V is the answer to the previous problem. After pouring the liquid, the height of the liquid L is 81 units. The total height H of the bottle is 120. The radius of the liquid r can be expressed in lowest form $\frac{\sqrt{a}}{b}$, where a and b are positive integers. What is the value of b?



ANSWER: $\boxed{9}$

$V\pi = L\pi r^2$. Thus $r = \sqrt{\frac{V}{L}} = \frac{\sqrt{19}}{9}$ and $b = 9$

5.3 Problem 3

A triangle has side lengths $\log_2 x$, $\log_4 x$, and b, where b is the solution to the previous problem. How many possible integer values of x are there?

ANSWER: $\boxed{59}$

Bound in both directions with triangle inequality, using $\log_4 x = \frac{1}{2}\log_2 x$. so $\frac{3}{2}\log_2 x > 9$, and $9 > \frac{1}{2}\log_2 x$. Then it would have been a very big number, with $2^{18} > x > 2^6$. However, during the contest, we changed the contest so that the answer to question 2 was 3, so it would actually be $2^6 > x > 2^2$, resulting in $63 - 4 = 59$ solutions for x.