2022 Team Math Attack Contest

Team Contest

December 17, 2022

Rules

1. You have 60 minutes to complete 20 problems (3 minutes/problem).

2. You start at 30 points, and every problem is worth 6 points (max score is 150).

3. You lose 1.5 points for every problem answered incorrectly, and get 0 points for every unanswered problem.

4. NO CALCULATORS. You will be disqualified if you use one.

5. EXACT VALUES ONLY (we want numbers like $\sqrt{2}$ and π)

6. Have fun and think hard!

Team Problems

1. A palindrome is a whole number whose digits read the same backwards as forwards. What is the smallest 5-digit palindrome?

2. How many positive odd integers are a factor of 2022?

3. During a road trip, Bill drove 580 kilometres using 16 litres of gas. His car drives 40 kilometres per litre on highways and 25 kilometres per litre in cities. How many kilometres of his road trip were on the highway?

4. How many triangles have a perimeter less than 15, one side with a length of 3, and the other two sides with prime number side lengths?

5. How many **different** arrangements of the letters "MATHATTACK" are possible? Don't forget, MATH-ATTACK counts as a arrangement too!

6. In a deck of 52 cards, if 13 of them are hearts, what is the probability that a randomly-dealt hand of 5 cards will contain at least one heart?

7. A triangle lies on a plane and has the coordinates $(4, 3)$, $(10, 11)$, and $(10, 8)$. What is the perimeter of the triangle?

8. What is the coefficient of the $a^8 d^2$ term of the expansion of $(2a + 5d)^{10}$?

9. A chip bag has 2 ketchup chips, 4 all-dressed chips, and 7 dill pickle chips. If you eat two chips, what is the probability of eating two different chips?

10. How many four digit numbers have exactly 9 divisors from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$?

11. There are 500 cups in a circle, and a ball is placed in every 11th cup starting with the 1st cup. Balls are placed in cups until another ball is placed in the 1st cup. Which cup is the 271st ball placed in?

12. If $\frac{1}{x} + x = 5$, then what is the value of $\frac{1}{\sqrt{x}} + \sqrt{x}$?

13. A box contains a collection of triangular and square tiles. There are 25 tiles in the box, containing 84 edges in total. How many triangle tiles are there in the box?

14. What is the sum of the digits of 999999^2?

15. $5040n$ has a square number of factors, where n is a positive integer. What is the sum of the digits of the lowest possible value of n?

16. Two circles with centres A and B with radii of length 8 and 9, respectively, are externally tangent. A line L drawn through B is tangent to circle A. A second line is drawn such that it is perpendicular to line L and tangent to circle A. The points are then labeled as shown below. What is DE^2?
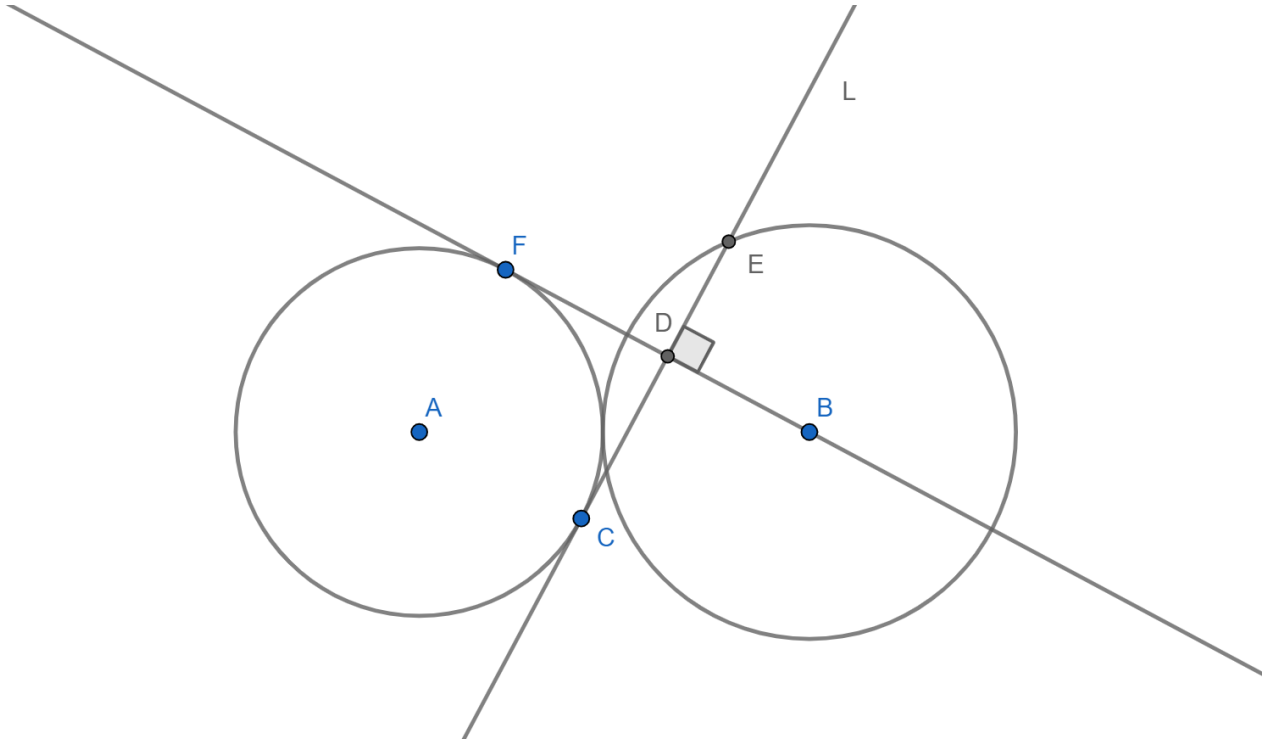
17. Define two functions $f(x)$ and $g(x)$ as follows:

$$f(x) = \sqrt{9x^2 + 4y^2 + 24x + 16y + 12xy + 16}$$
$$g(x) = 2x - 2y + 2xy - 1 - x^2 - y^2$$

There is one unique set of real numbers x and y such that $f(x) = g(x)$. What is $3x + 2y$?

18. $\triangle ABC$ with side lengths $AB = 2\sqrt{2}$ and $BC = 2\sqrt{2}$ is inscribed in a circle with diameter 4. A chord of length $2\sqrt{3}$, parallel to the chord AC, divides the triangle into two shapes: a trapezoid and a triangle. What is the area of this new triangle?

19. There is a polynomial $P(x) = x^3 + Ax^2 + Bx + C$, where all coefficients are rational numbers. If $P(2 + i) = 0$ and $B = 13$, what is the value of A, where $i = \sqrt{-1}$?

20. Ashachu starts adding and gets bored of the small sums they get from adding positive integers together, so he starts turning the $+$ signs into $\times$'s. For instance:

$$5 + 3 + 2 = 10 \longrightarrow 5 \times 3 \times 2 = 30$$

He finds that the expression they create often has a greater value than the original sum. Let N be the greatest value Ashachu can make using an expression with sum 2022. What are the last two digits of N?