2022 Team Math Attack Contest

Relay Round

December 17, 2022

Rules

1. You have 60 minutes to complete 5 problems with 3 parts (A, B, C) each (4 minutes/part).
2. You get 6 points for Part A, 8 points for Part B, and 10 points for Part C
3. You start at 30 points, and every problem is worth 24 points (max score is 150).
4. You lose 1.5 points for every problem answered incorrectly, and get 0 points for every unanswered problem.
5. NO CALCULATORS. You will be disqualified if you use one.
6. EXACT VALUES ONLY (we want numbers like $\sqrt{2}$ and π)
7. Have fun and think hard!

Jeremy has to head to school each day. His school is 2100m from his house. The first day, he goes by bike, at 20km/h, the next day by bus at 30km/h, and the third day by car at 60km/h.

Problem 1.1

How long does it take for Jeremy to get to school when he is biking? Express this answer in minutes with decimals.

Problem 1.2

How long does it take for Jeremy to get to school when he is bussing? Express this answer in minutes with decimals.

Problem 1.3

What is his average speed throughout all 3 commutes in km/h? Express this answer in km/h.

Problem 2.1

Today is Thursday, September 8, 2022. What day of the week will it be 100 days from today? If it is a Monday, use $n = 1$ for the next problem. If it is a Tuesday, use $n = 2 \ldots$ If it is a Saturday, use $n = 6$. If it is a Sunday, use $n = 7$.

Problem 2.2

A magic candle is n centimeters long, where n is the answer to the previous problem. For every cm of the candle that burns, an additional 2 cm is added to the candle. In addition, for every cm of the candle that burns, the next cm of the candle takes one minute longer to burn. Assuming that the first cm of the candle takes 1 minute to burn, in how many minutes will the candle be double its original length? Let this number be m, which we will use in the next problem

Problem 2.3

What is the total number of distinct rectangles with integer side lengths that have an area of m, where m is the answer to the previous problem?

Aphelios has 5 different weapons: rifle, scythe, cannon, flamethrower, and chakram. He goes through one full cycle when he has used all his weapons once. After a cycle is completed, Aphelios returns to the first weapon used and continues to use his weapons in the same order.

Note: The order is circular, and rotations are considered the same. For example, rifle, scythe, cannon, flamethrower, and chakram are the same as scythe, cannon, flamethrower, chakram, and rifle.

Problem 3.1

If he uses one weapon at a time, how many weapon orders are possible?

Problem 3.2

If he wants to use his scythe and chakram consecutively, how many weapon orders are possible?

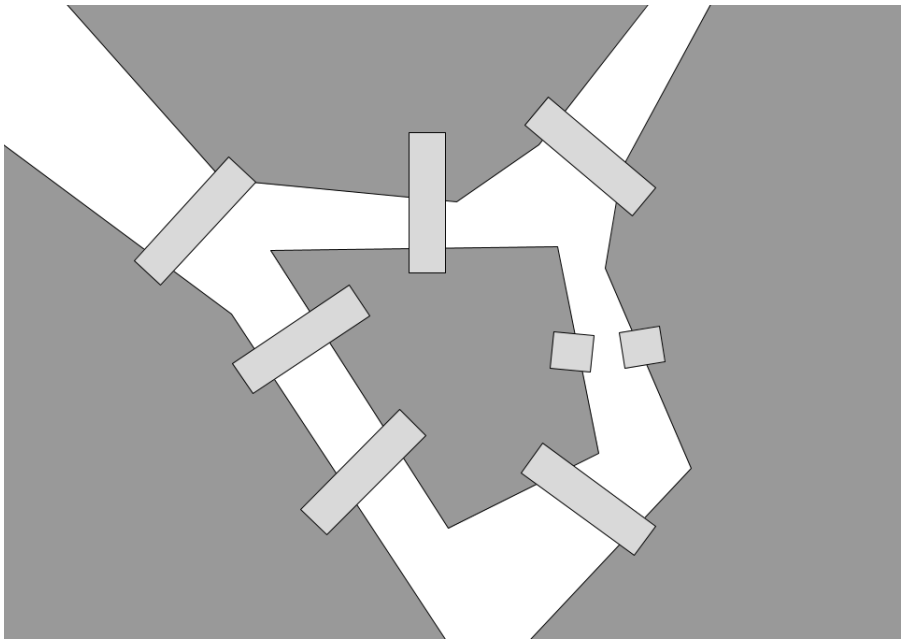
Problem 3.3

What is the probability that cannon and chakram are not used consecutively?

Problem 4.1

People have been disappointed in the infrastructure of Gammasberg for a long time. Ashachu is attempting to beat the gym challenge, which is to go across all of the bridges once. He is finding difficulty with this, so Ashachu has tactically removed a bridge, as shown below. How many ways are there for Ashachu to cross on all of the remaining bridges exactly once now if he starts from the top island?

Note: Ashachu may not swim or remove more bridges. He can move around on individual landmasses as much as he can. The landmasses are not connected outside the image.



Problem 4.2

Let m be the answer to the previous problem. In the magical world of Mathamon, Pi and Ashachu are in a battle against Mathikyu with 100 health. Each time Ashachu attacks, Ashachu will reduce Mathakyu's health by m. After every 5th time Ashachu attacks, Mathakyu's current health will increase by exactly 50%. How many attacks does it take for Ashachu to defeat Mathakyu?

Problem 4.3

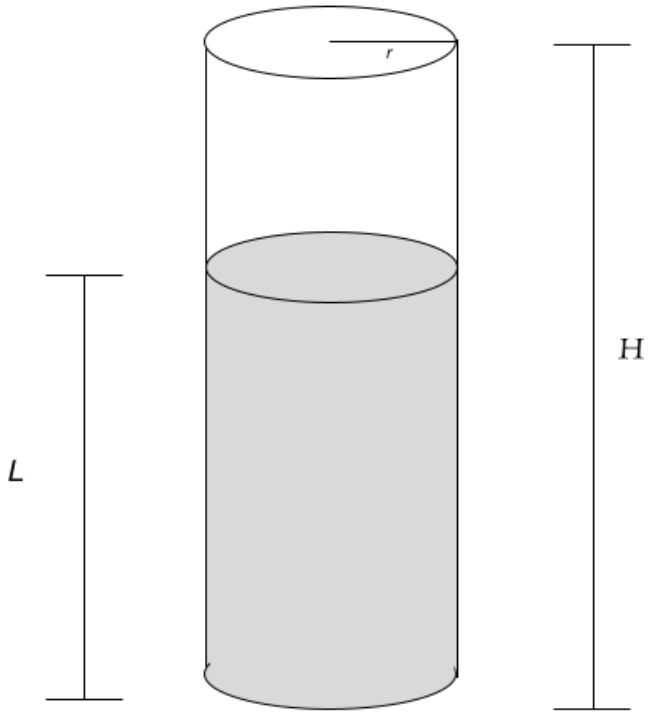
Let n be the answer to the previous problem. Ashachu is putting his badges in holders after winning the Pokemon league. He can put his badges in holders of 6 or 11. If he puts them in holders of 6, he will have 5 badges left over, and if he puts them in holders of 11, he has n badges left over. What is the smallest number of badges he can have?

Problem 5.1

A clock strikes at regular intervals throughout the day, starting at exactly 12 o'clock. Between 1 and 2 o'clock, it strikes 6 times. What is the greatest number of times it can strike in the three hours from 1 o'clock to 4 o'clock? Let this number be V.

Problem 5.2

Ryan pours an unknown amount of a mystery liquid into an upright cylindrical bottle. Assume that the liquid's volume is $V\pi$ cubic units, where V is the answer to the previous problem. After pouring the liquid, the height of the liquid L is 81 units. The total height H of the bottle is 120. The radius of the liquid r can be expressed in lowest form $\frac{\sqrt{a}}{b}$, where a and b are positive integers. What is the value of b?



Problem 5.3

A triangle has side lengths $\log_2 x$, $\log_4 x$, and b, where b is the solution to the previous problem. How many possible integer values of x are there?