# 2021 Team Math Attack Contest

Team Contest

December 18, 2021

Part 1

1. How many different four-letter sequences can be made by rearranging the letters of "MATH"?

2. In poker, a "Four of a Kind" is a hand of 5 total cards, consisting of 4 cards with the same rank but different suits, and a 5th card that could be anything. How many Four of a Kind hands are possible in a standard deck of 52 cards?

3. What is the middle digit of 1111111^2?

4. My birthday is in the latter half of the year and in 2019 it happened to be on a Tuesday. When will be the next year my birthday occurs on a Wednesday?

5. Richard has 4L of a 20% lemonade drink, and 4L of a 70% lemonade drink. How much of the 70% lemonade drink should Richard combine with the 20% lemonade drink to create 3L of a 60% lemonade drink?

Part 2

6. If $a - 7 = b + 3 = c - 2 = d + 22$, which one of a, b, c, d is largest?

7. A quadrilateral is built with each side being tangent to a circle inscribed in it. If two opposite sides are of length 15 and 20, and a third side is length 28, what is the length of the last side?

8. The geometric mean of 2 numbers is the square root of the product of the numbers. The arithmetic mean of 2 numbers is their sum divided by 2. The sum of 2 positive integers a, b is 20. If the geometric mean of a, b is greater than or equal to the arithmetic mean of these 2 numbers, then what is ab?

9. How many 0's are at the end of 100!? (A factorial is the product of all positive integers less than or equal to a given positive integer. For example, $7! = 1 * 2 * 3 * 4 * 5 * 6 * 7$).

10. In triangle $\triangle CBA$, $\overline{CD}$ and $\overline{AE}$ are medians, and $\overline{FC}$ is parallel to $\overline{AB}$. The area of $FCGE$ is 7, and the area of $EGDH$ is 11. Compute the area of triangle $\triangle CBA$.
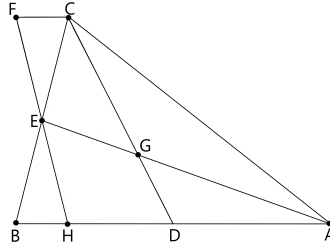
Part 3

11. How many ways are there to travel from A to B, and then to C if you can only move down or to the right?
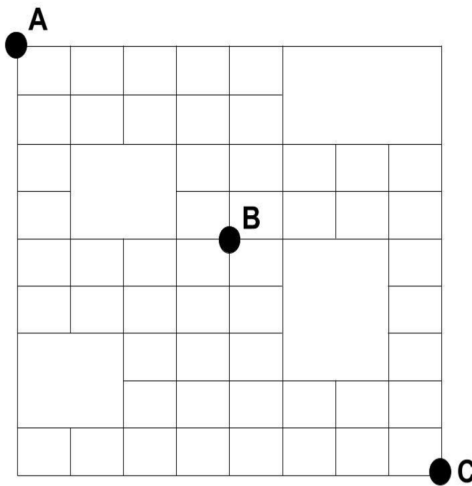
12. The Fibonacci sequence is the series of numbers:

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ ...$$

where the next number in the sequence is found by adding up the two numbers before it.

Every positive integer n can be expressed uniquely as a sum of distinct non-consecutive Fibonacci numbers. This is known as the Zeckendorf representation of n. For example, here is the Zeckendorf representation of 54:

$$54 = 2 + 5 + 13 + 34$$

The sum of the digits in the Zeckendorf representation of 54 is $2 + 5 + (1 + 3) + (3 + 4) = 18$. What is the sum of the digits of the Zeckendorf representation for 2021?

13. Four distinct integers are to be chosen from the set 1, 2, 3, 4, 5, 6, 7, 8 and placed in some order in the top row of boxes in the diagram. Each box that is not in the top row then contains the product of the integers in the two boxes connected to it in the row directly above. Determine the number of

ways in which the integers can be chosen and placed in the top row so that the integer in the bottom box is 3000.
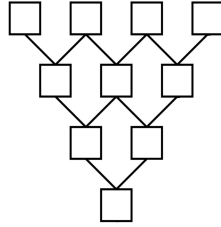
14. Richard, Henry, and Michelle have 14 books in total. They each have at least one book, each one has a positive number of books, and are initially only aware of their own number of books.

    Richard: I can tell using the number of books I have that the number Henry and Michelle have are not the same

    Henry: I can tell using the number of books I have that there is no way any of us have the same number of books.

    Michelle: After hearing what Richard and Henry said, I can use my number to figure out how many books we each have.

    How many books do they each have?

15. A machine will do the following 3 steps if inputted with a number x:

    1. Replace the first two digits of the number with the sum of the two first digits.

    2. Replace the last two digits of the number with the sum of the last two digits.

    3. Repeat until only a single number remains.

    Ex. If the number 98123 is inputted into the machine, it will change as such:

    $$98123 \rightarrow 17123 \rightarrow 1715 \rightarrow 815 \rightarrow 86 \rightarrow 14 \rightarrow 5$$

    What will be the result if 20212022202320242025202620272028029 is inputted into the machine?