# 2020 Team Math Attack Contest

Team Contest

December 12, 2020

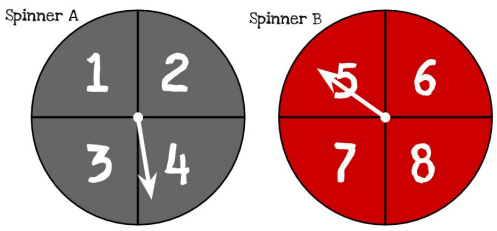
## 1 Easy

1. A 1 GHz processor can run 1 billion operations in 1 second. How many operations can it run in 1 hour?

   ANSWER: $\boxed{3600000000000}$

   1 hour is equal to 60 minutes, which is equivalent to $60 \times 60 = 3600$ seconds, so in total we have $\boxed{3600000000000}$ operations.

2. Joe has two spinners labeled A and B. As shown, Spinner A is split into four equal sections, labeled with 1, 2, 3, and 4, while Spinner B is split into four equal sections labeled with 5, 6, 7, and 8. To create a two digit number, he spins Spinner A to obtain the value of the ten's digit, and then spins Spinner B to obtain the one's digit. What is the probability that the number Joe gets is a multiple of 3?




   ANSWER: $\boxed{\frac{3}{8}}$

   The only multiples of 3 with 1, 2, 3, or 4 as the tens digit and 5, 6, 7, or 8 as the ones digit are 15, 18, 27, 36, 45, and 48. There are $4 \times 4 = 16$ possible numbers total, each with equal probability, and so we get $\frac{6}{16} = \boxed{\frac{3}{8}}$.

3. Point A(2,5) and B(6,5) and C(6,8) form a triangle on a grid. Calculate the product of the side lengths of the triangle.

   ANSWER: $\boxed{60}$

   We have $AB = 4$, $BC = 3$, and $AC = 5$, so $3 \times 4 \times 5 = \boxed{60}$.

4. In 2019, 72 students were enrolled in the geometry class. 59 were enrolled in the algebra class. If 34 were enrolled in both, how many students in total were enrolled in these math classes?

   ANSWER: $\boxed{97}$

   There are $72 - 34 = 38$ students only in geometry, $59 - 34 = 25$ students only in algebra, and 34 students in both, for a total of $38 + 25 + 34 = \boxed{97}$ students in total.

5. Alex is planning on starting a company called Cai Foundations, and he had \$2673 before borrowing some money from the bank. After borrowing money, he spent $\frac{1}{2}$ of his money on company resources, $\frac{1}{4}$ of his money on renting a facility, and $\frac{1}{6}$ of his money on hiring new workers. Afterwards, he had exactly the same amount of money remaining as the amount that he borrowed from the bank. How much money did he borrow from the bank for his startup?

   ANSWER: $\boxed{243}$

   Let $x$ be the amount of money that Alex borrowed.

   Adding up the fractions, we see that he spent $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$ of his money. That means that he has $\frac{1}{12}$ of his money remaining.

   Since this is equal to the amount he borrowed,

$$\begin{aligned} x &= \frac{1}{12}(2673 + x) \\ 12x &= 2673 + x \\ 11x &= 2673 \\ x &= \boxed{243} \end{aligned}$$

# 2 Medium

6. The sum of 2025 and a non-zero perfect square results in a new perfect square. Find the smallest possible value of the new perfect square.

   ANSWER: $\boxed{2601}$

   Let the sum be $x^2$ and the addend be $y^2$, such that $2025+y^2 = x^2$. Then, $2025 = x^2-y^2 = (x-y)(x+y)$. Let $m = x - y$ and $n = x + y$ such that $x = \frac{m+n}{2}$ and $y = \frac{n-m}{2}$. Since the prime factorization of 2025 is $3^4 \times 5^2$ and we want to make $m + n$ as small as possible, we have $m = 75$ and $n = 27$ so that $y = 24$ and $x = 51$, so $x^2 = \boxed{2601}$.

7. Stefanie flips a coin five times and records the results. What are the chances that she lands more than one tail? Enter your answer as a fraction in lowest terms p/q.

ANSWER: $\boxed{\frac{13}{16}}$

To solve this question, we find the number of ways she can land one tail or less and subtract this from the total.

# of ways to land 0 tails: 1 (all coins are heads)

# of ways to land 1 tail: 5 (any one of the five coins might be tails and the rest will be heads)

Total: $2^5 = 32$

So our final probability is $(32 - 5 - 1)/32 = 26/32 = \boxed{\frac{13}{16}}$.

Use the following information for questions 8 and 9.

| Reverse polish notation is a different way of writing mathematical expressions. Instead of going in between the numbers, the operators come directly after. That is, "1 3 +" is equivalent to 1 + 3, and "4 3 6 / -" is equivalent to "4 (3 6 /) -" which is equivalent to $4 - \frac{3}{6} = 3.5$. |
|---|

8. Evaluate "4 1 2 / 1 3 / - /".

   ANSWER: $\boxed{24}$

$$\begin{aligned} 4\ 1\ 2\ /\ 1\ 3\ /\ -\ / &= 4\ ((1\ 2\ /)\ (1\ 3\ /)\ -)\ / \\ &= \frac{4}{\frac{1}{2} - \frac{1}{3}} \\ &= \frac{4}{\frac{1}{6}} \\ &= \boxed{24} \end{aligned}$$

9. Using the numbers 1, 3, 4, and 6 exactly once each in any order, you can form the number 24 using the elementary operations of

   (a) Addition (+)

   (b) Subtraction (-)

   (c) Multiplication (*)

   (d) Division (/).

   Express your solution in the form $abcdijk$, where $a$, $b$, $c$, and $d$ form some ordering of 1, 3, 4, and 6, and $i$, $j$, and $k$ are each one of $+$, $-$, $*$, or $/$, corresponding to the operations above, such that the expression "a b c d i j k" equals 24 using reverse polish notation.

   ANSWER: $\boxed{6134/-/}$

   $\frac{6}{1-\frac{3}{4}} = \frac{6}{\frac{1}{4}} = 24$. Writing this in reverse polish notation, we have "6 (1 (3 4 /) -) /". Removing the brackets, we have $\boxed{6134/-/}$.

10. How many different ways can you rearrange the letters in "MATHATTACK" if 1) the vowels must be together and 2) the letters C and K must NOT be together?

ANSWER: $\boxed{5040}$

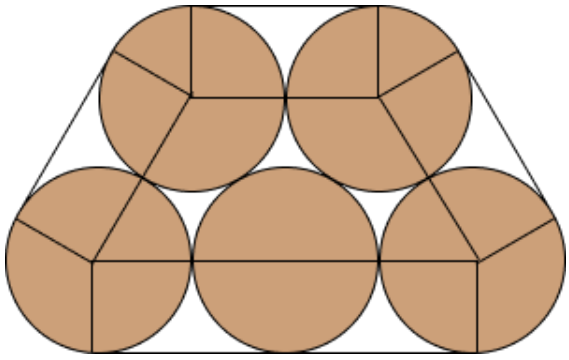
Since the vowels are together, we can treat the A's as one group. Since we are left with 8 "items", M T H T T C K (AAA), and there are 3! ways of arranging the 3 T's, we have $\frac{8!}{3!}$ ways to arrange these items.

Now, we want to subtract the number of cases where C and K are together. Treating them as one group, we have M T H T T (CK) (AAA) for a total of 7 groups, which gives us $\frac{7!}{3!}$ such cases since we are counting each ordering of the T's 3! times. However, since CK and KC are both valid orderings, we multiply this by 2 to get $2 \times \frac{7!}{3!}$. Subtracting this from the total, we have $\frac{8!}{3!} - 2 \times \frac{7!}{3!} = (8-2) \times \frac{7!}{3!} = 6 \times 7 \times 6 \times 5 \times 4 = \boxed{5040}$.

# 3 Hard

11. Five circular logs of the same size are tied together using a piece of rope as shown. If the length of the rope is $85 + 17\pi$ inches, what is the radius of each log in inches?

ANSWER: $\boxed{8.5}$



Let the radius of each log be $r$. Connect the origins of the circles to each other and to the surrounding lines as shown. We see that adding all of the curved portions simply gives us the circumference of one of the circles, or $2\pi r$, while the remaining sidelengths add to give us $2r + 2r + 2r + 4r = 10r$. In total, we have $2\pi r + 10r = 17\pi + 85$, so $r = \boxed{8.5}$.

12. The first term of a sequence is AB. To create each subsequent term, each A in the term before is replaced with AB, and each B in the term before is replaced with BAB. The first three terms of the sequence are below. Find the number of B's in the eighth term.

    (a) AB

    (b) ABBAB

    (c) ABBABBABABBAB

ANSWER: $\boxed{987}$

We see that the number of A's in a line is equal to the number of A's plus the number of B's in the previous line, and that the number of B's in a line is equal to the number of A's in the previous line

plus twice the number of B's in the previous line. Thus, using a calculator we can quickly calculate that there are $\boxed{987}$ B's in line 8.

13. Varth Dader is building the biggest Steath Dar ever seen in the galaxy. To do this, he instantaneously creates 2020 self-replicating drones. Each hour, each drone produces 2019 replicas of itself.

    After 1 week of replication, Varth needs to hide the drones from the Emperor. He has drone docks which can fit exactly 2021 drones each. If he hides as many drones as possible in his docks, how many drones will be left over?

    ANSWER: $\boxed{2020}$

    He starts with 2020 drones. After 1 hour, each of these drones creates 2019 drones. That is to say, each drone from the start is equivalent to 2020 drones after an hour. So, after $n$ hours, the number of drones is $2020^{n+1}$.

    Now, in 1 week, there are 7 days * 24 hours/day = 168 hours. This means at the end of the week, there are $2020^{169}$ drones.

    Note that 2020 divided by 2021 has a remainder of 2020, which is equivalent to -1. That means that when we remove drones in boxes of 673, there will be $(-1)^{169}$ drones left over, which is equivalent to -1 since the exponent is odd. Therefore, we will have $2021 - 1 = \boxed{2020}$ drones left over.

14. Suppose that you're an eccentric chef at a restaurant! For dinner, you decide to prepare some soup. In front of you, you arrange some number of cans of Campbell's soup in a grid on the floor with $m$ rows and $n$ columns, where both $m$ and $n$ are less than 1000 because your restaurant is not large enough to house more soup and $m$ is less than $n$. You soon realize that you have too many cans of soup, so you remove 32 rows and 32 columns from your grid of cans of soup. You are now left with 2021 cans of soup, the perfect amount for today's restaurant orders! Knowing this, find $m + n$.

    ANSWER: $\boxed{154}$

    After removing the rows and columns, we have that $(m - 32)(n - 32) = 2021$. Since 2021 the prime factorization of 2021 is $43 \times 47$ and m is less than n,

    $m - 32 = 43$, so $m = 75$. $n - 32 = 47$, so $n = 79$ and $m + n = 75 + 79 = \boxed{154}$.

15. A group of soup-loving pirates happened to stumble upon a giant grid of soup cans. The first pirate said, "We should split the cans all equally, and there will be 2 cans left over." The second pirate said, "We can kick out Johnny because they didn't swab the deck yesterday, and then we'll be able to split the cans evenly." The third pirate, who was practicing to be a mathematician, said, "In fact, we can take out any number $n$ pirates except for $n = 3$ and $n = 5$ and still end up with an even split."

    What is the least number of pirates in the group?

    ANSWER: $\boxed{10}$

    Let $p$ be the least number of pirates in the group.

    We know that $p$ is greater than 5 because p-5 must be greater than 0.

    If $p = 6$, then the cans must not be divisible among $6 - 5 = 1$ pirate, which is impossible.

    If $p = 7$, then the cans must not be divisible among 2 or 4 pirates. The cans, however, will be divisible among 6 pirates, making it impossible.

If $p = 8$, the cans must not be divisible among 3 or 5 pirates, but must be divisible among 1, 2, 4, 6, and 7 pirates. This means that the number of cans has to be a multiple of 2*2*3*7 = 84. However, 84 is congruent to 4 (mod 8) instead of 2 (mod 8) like the first pirate said. There is no multiple of 84 that will be 2 (mod 8), since all multiples of 4 (mod 8) are either 4 or 0 (mod 8). Therefore, it is impossible to have 8 pirates.

If $p = 9$, the cans must not be divisible by 4 but must be by 8, making it impossible.

If $p = 10$, the cans must not be divisible by 5 or 7 but must be divisible by everything else. Using the same logic as $p = 8$, we find that the number of cans must be a multiple of $2 \times 3 \times 2 \times 3 = 36$, which is congruent to 6 (mod 10). If we multiply this by 2, we will end up with 72 cans, which satisfies our condition of 2 (mod 10). Therefore, the minimum number of pirates is $\boxed{10}$.