

# TEAM MATH ATTACK CONTEST

## PART A

Saturday November 30th, 2019

Time: 30 minutes

**Calculators are NOT allowed.** Students are not allowed to leave the room during testing time.

### Instructions

1. Do not open the contest paper until you are told to do so.
2. You may use rulers, compasses, protractors, and graph paper for rough work, but all problems can be solved without additional aid.
3. Write your team name, member names, and all corresponding schools at the top of the response sheet. Print clearly. When you are finished, submit the exam booklet with your answer sheet attached or tucked inside.
4. A box to place your answer follows each question on the response sheet. To receive full marks, you must simply write your answer in the appropriate blank space. Use exact values (i.e.  $\sqrt{3}$ , or  $\pi + 2$  etc.) or rounded answers to the thousandths decimal place (i.e. 324.237).
5. Each correct answer in Part 1 is worth 1 point. Each correct answer in Part 2 is worth 2 points. Each correct answer in Part 3 is worth 3 points. There is no penalty for an incorrect answer. Partial marks will not be awarded.
6. When your supervisor tells you to begin, you will have 30 minutes of working time.

**Good luck and have fun!**



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# Easy

## Problem 1

Find the simplified value of

$$\frac{9 + 27}{18 + 54}$$

Note: The simplified value is either a decimal or a fraction  $\frac{a}{b}$  such that  $a, b$  are not both simultaneously divisible by the same prime.

## Problem 2

How many odd whole numbers (i.e. 1, 3, 5, 7, ...) lie strictly between  $2^3$  and  $2^4$ ?

## Problem 3

The perimeter of a triangle  $\triangle ABC$  is 34. If  $\angle ABC = \angle ACB$  and  $BC = 14$ , what is the length of  $AC$ ?

## Problem 4

A regular polygon has 6 sides. Regular means that all side lengths and angles are the same. Let  $x$  be the value of each interior angle, measured in degrees. Find the value of  $x$ .

## Problem 5

Andrew is trying to learn a new language called Eсениhc. Apparently the number system for Eсениhc is very similar to English, and each digit from 0-9 is represented by a character. For example, 十五 = 15. In English numerals (i.e. 1,2,3, etc.), find the value of

$$十五 + 五十$$

## Medium

### Problem 6

Jack wants to buy Math-bits for him and his friends. Him Tortons sells 10, 15 and 20 Math-bits for 3\$, 3.50\$, and 5\$ respectively. What is the least amount of money he will have to spend to evenly distribute his Math-bits, without any leftovers, among himself and 5 friends?

### Problem 7

Jason places 12 balls in a bag. Each ball is one of  $n$  colours. If Jason takes out 5 balls from the bag, he will always have a pair of the same colour. What is the maximum value for  $n$ ?

### Problem 8

What is the units/ones digit of the following number?

$$2019! + 2020!$$

Note:  $n!$  is 'factorial notation'.  $n!$  is defined as the product of every positive integer from 1 to  $n$ . For example,

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

### Problem 9

Let  $P$  be the largest prime number strictly less than 100. Let  $Q$  be the smallest prime number strictly greater than 100. What is the value of  $P \cdot Q$ ?

### Problem 10

Alex had no friends before and finally met Andrew on the 1st of June. They plan to hangout to further their friendship, but they realize their work breaks do not align. Alex gets a work break every 9th day and Andrew gets a work break every 6th day. If they start work on June 1st, and they hangout every time their work breaks align, when will their 4th hangout be?

# Hard

## Problem 11

Find two positive integers  $(x, y)$  such that:

$$101 + x^2 = y^2$$

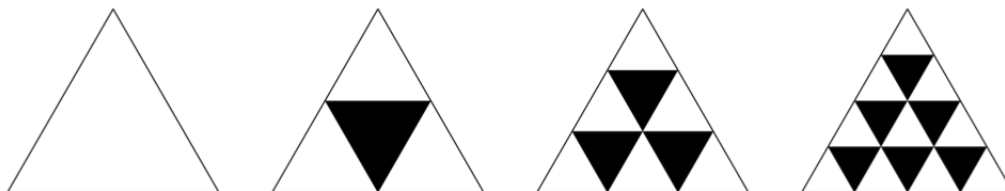
## Problem 12

$$\begin{array}{r} J \quad E \quad R \\ + \quad R \quad E \quad E \quad E \\ \hline R \quad I \quad R \quad I \end{array}$$

Given that each letter represents a distinct digit from 1 - 9 and  $I = 8$ , find the value of  $J + E + R + R + I$ .

## Problem 13

Consider the following sequence of triangles. The  $n$ -th triangle has  $n$  rows of smaller triangles. All smaller triangles are congruent to each other.



In the fifteenth triangle, what is the ratio between the area of the shaded region and the area of the non-shaded region?

## Problem 14

A circle with radius  $r$  has 2019 distinct points on the circumference. Two points,  $A$  and  $B$ , are chosen at random from these 2019 points and a chord is drawn from  $A$  to  $B$ . Of the 2017 remaining points, two points,  $C$  and  $D$ , are then chosen at random and a chord is drawn from  $C$  to  $D$ . What is the probability that these two chords intersect?

## Problem 15

Consider the function

$$g(x) = \frac{5x^4 + x^3 + 2x^2 + xy + y^2 + 1}{x + y + 1}$$

It is known that  $y = 1$ . Let  $P$  be the product of all of the possible integer values  $x$  such that  $g(x)$  is a positive integer. Let  $f(a) = (a^2 + 1)^P$ , where  $a$  is any real number. Find the value of

$$f(1) - 2f(3) + 3f(5) - 4f(7) + 5f(9)$$

**Team Name:**

\_\_\_\_\_

**Team Members:**

1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

**School:**

\_\_\_\_\_

**Part:** | A |

Please do not write anything in the columns labeled "Mark" or under "GRADER'S USE ONLY".

	Part 1	Mark
1.		
2.		
3.		
4.		
5.		

	Part 2	Mark
6.		
7.		
8.		
9.		
10.		

	Part 3	Mark
11.		
12.		
13.		
14.		
15.		

**GRADER'S USE ONLY**

Grader #1

1   ×   \_\_\_\_\_ = \_\_\_\_\_

2   ×   \_\_\_\_\_ = \_\_\_\_\_

3   ×   \_\_\_\_\_ = \_\_\_\_\_

Grader #2

1   ×   \_\_\_\_\_ = \_\_\_\_\_

2   ×   \_\_\_\_\_ = \_\_\_\_\_

3   ×   \_\_\_\_\_ = \_\_\_\_\_

**Final Score:** \_\_\_\_\_ / 30