Part 1: Easy

Problem 1

Jerry (a male) has 2 more sisters than he has brothers. If the total number of children in the family is divisible by 9, what is the remainder when the number of brothers in the family is divided by 3?

Problem 2

How many ways can you rearrange the letters in the word "CLOSENESS"?

Problem 3

Determine the number of times the digit 7 appears in the set $\{1, 2, \ldots, 99, 100\}$.

Problem 4

Find the remainder of 6^{2018} upon division by 10.

Problem 5

In the set $\{49, 63, 70, 77, 126\}$, which number is the average of the other four numbers?

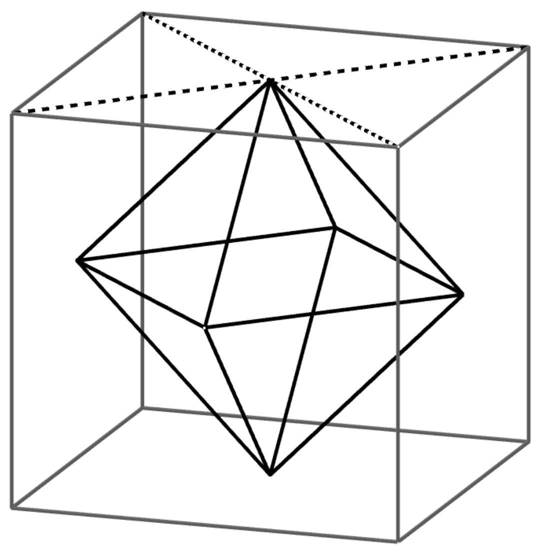
Part 2: Medium

Problem 6

Find the units digit of $(3^{100})(2^{101})$.

Problem 7

The centre of a square is the point of intersection between the two diagonals. Given a cube of side length 1, the centres of the 6 faces form the vertices of an octahedron. Find the volume of this regular octahedron.



Problem 8

Given that $\frac{15!}{10^k} = q$ where k, q are both positive integers, determine the maximum value of k.

Problem 9

Let x be some integer. What is the sum of all solutions to the following equation,

$$((x-3)(x-4)+1)^{(x+10)(x-6)} = 1$$

Problem 10

In the given 7x7 grid, there is a diamond hidden in exactly 10 of the 49 squares. The numbers indicate how many adjacent squares (sharing a side or corner) contain a diamond. No square with a number also contains a diamond. Indicate the locations of these 10 diamonds in the diagram **attached to response sheet**.

		5		4	1	
	2					
				4	2	
	0			3		1
				2	1	
			2			

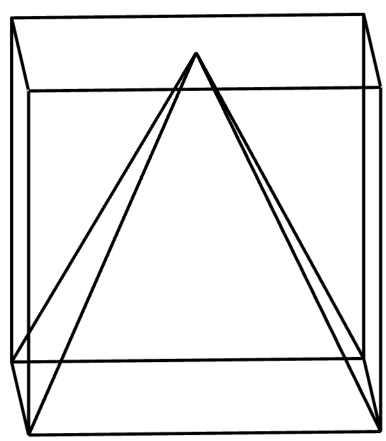
Part 3: Hard

Problem 11

Michael and Chan are running down a 400 m circular race track at constant but different speeds. They start at the same point and run in opposite directions. Michael runs at 1.5 m/s and they pass by each other every 1m 40s. How fast does Chan run in m/s?

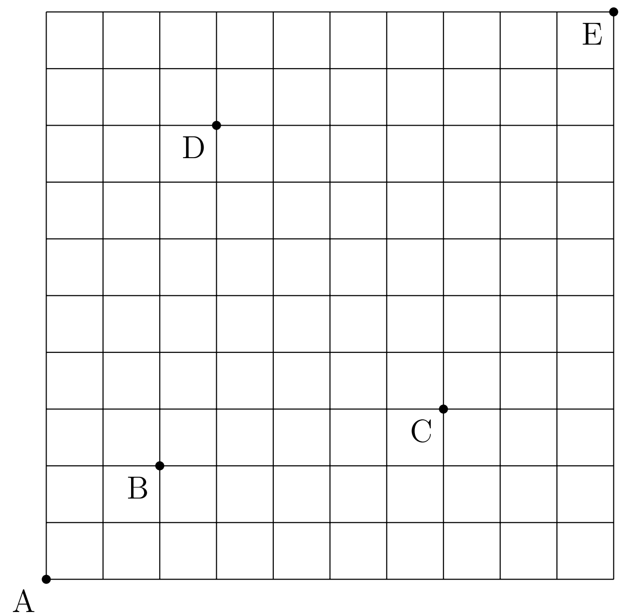
Problem 12

Bob has a square pyramid inside a cube that shares the same base as the cube and with the upper vertex touching the centre of the top face of the cube. Bob then decides to scale everything by a factor of 4. If the surface area of the pyramid increased by $\frac{303}{4}\text{cm}^2$, what is the side length of the cube after scaling?



Problem 13

Jerry Li wants to move through 4 of these 5 marked points, but he can only move upwards or to the right. How many ways can Jerry fulfill these conditions?



Problem 14

The expression $343^9 \cdot 216^{12} \cdot 49^7 \cdot 2401^5 \cdot 7^6 \cdot 36^{10}$ can be written in the form $a^b \cdot c^d$, where a, b, c, d are positive integers,[1] $b < d$, and a, c are in the lowest bases possible. Find the units digit of $a^{2012} + b^{2014} + c^{2016} + d^{2018}$.

[1] On the original contest, there was a typo in the expression statement. As a result this problem was omitted.

Final Proof

Problem 15

The terms of the sequence $\{a_i\}$ are defined as follows

i. $a_{2n} = a_n$.

ii. For all odd n. $a_n = a_{n-1} + a_{n-3} + \ldots + a_2 + a_0$

Given $a_0 = 1$, prove that, for any $k \geq 2$, a_{3k} is always even and a_{3k+1}, a_{3k+2} are always odd.