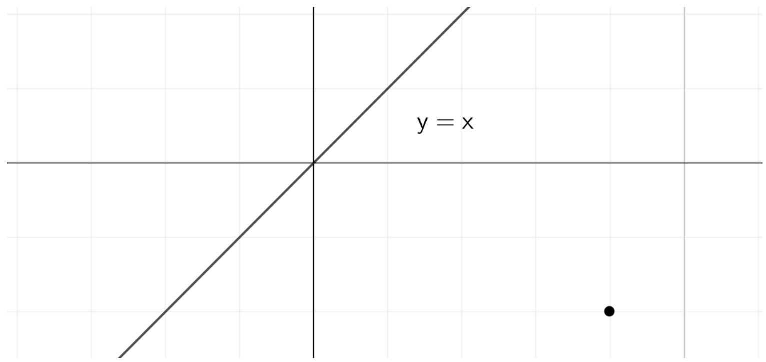
Part 1: Easy

Problem 1

The point (4, -2) is reflected over the x-axis. The resulting point is then reflected again over the line with equation $y = x$. What are the coordinates of the new point?



Problem 2

Find the sum of the first 10 positive powers of 2.

Problem 3

The mean of the 9 data values

$$20, 80, x, 60, 15, 35, 95, 100, 70$$

is equal to $x - 5$. Find the value of x.

Problem 4

Monika has three distinct dogs in a line. A golden retriever, a pug, and a dalmatian. She has four different collars. How many ways can she arrange her dogs with exactly one collar each?[1]

Problem 5

Let a, b be the remainders of 12345678987654321 upon division by 9 and 11 respectively. Find $a + b$.

[1] Clarified from original problem

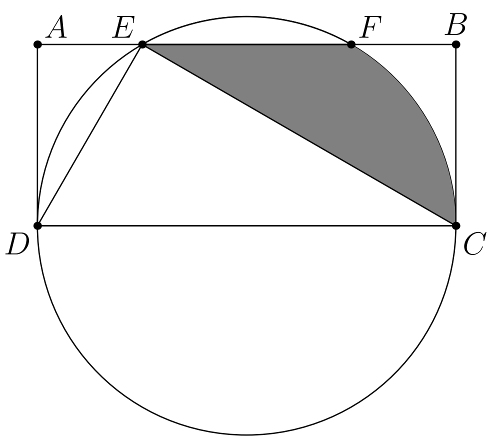
Part 2: Medium

Problem 6

Jack can create an entire math contest in 10 days. Andrew can create the same math contest in 8 days. Assuming they always work at the same rate, how many days will it take for them to create one math contest if they work together?

Problem 7

In rectangle $ABCD$, point E is chosen on side AB such that $\triangle DEC$ is a right triangle. The circumcircle of $\triangle DEC$ intersects AB at F. Given $\angle DCE$ is $30°$, find the ratio of the area of the region contained inside EC, FE and the arc FC to the area of $\triangle DEC$.



Problem 8

10! can be expressed as $2^a \cdot 3^b \cdot 5^c \cdot \ell$ where ℓ is not divisible by 2, 3, or 5. Determine $a + b + c$.

Problem 9

Levin Kin is competing in a math competition with 4 other contestants. Each contestant plays 3 games head-to-head with each other contestant, playing 12 games overall. The contestant with the most wins overall wins the entire tournament. How many games does Levin need to win in order to guarantee he will win the competition, without anyone else tying him for first?

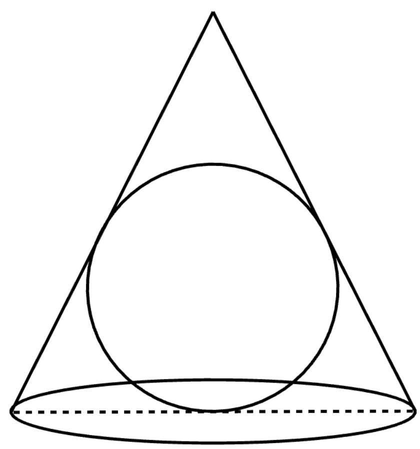
Problem 10

John the Zheng boat was rowing downstream at a speed of 17 km/h. After 35 minutes, he turned around and rowed back upstream the same distance. If his average speed was 10 km/h, find the speed at which he rowed upstream.

Part 3: Hard

Problem 11

What is the volume of the sphere inscribed inside a cone with base radius 10 cm and height of 15 cm?



Problem 12

Find the number integers x such that $\dfrac{x^2+3x-42}{x-3}$ is an integer.

Problem 13

How many integers from 1-1000 are spelt with 21 letters? (Ex. four hundred twenty four).

Problem 14

Determine two positive integers a, b such that $a^2+b^2 = 10201$.

Final Proof

Problem 15

The integers from 1 to 100 are written on a whiteboard. Aaron circles 51 of these integers. Prove there must be a circled integer that is the multiple of another circled integer.